

# Getting Money Out of Politics... And Into Charity

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## 1. Introduction

In 2016, over 1.8 billion dollars were raised and spent by the campaigns of Hillary Clinton and Donald Trump [1]. This is inefficient, in the sense that the money is spent adversarially: money spent by the Clinton campaign is intended to convince voters to elect Clinton and money spent by the Trump campaign is intended to convince voters to elect Trump. One might naïvely expect that increased spending by both campaigns could cause voters to make a more informed choice, but this view is not supported by evidence. As a matter of fact, there is no evidence that campaign spending in general elections has any effect if both candidates are well-known, as Clinton and Trump were [4].

This suggests a potential pareto improvement: if two supporters of opposing candidates wish to donate \$100 to their preferred candidate, they could instead mutually agree not to donate. The most glaring problem here is that it requires trust between the donors: one could imagine the donors making this agreement and one donor donating anyway without informing the other. However, a slight modification to this idea is more robust: both donors agree to donate their \$100 to charity. In this case, making and then breaking the agreement costs more money than a donor initially intended to spend.

The macro version of this agreement is an online platform to which supporters of both parties can donate. Periodically (say, every week), the platform donates matched amounts of money to charity and donates the rest to the candidate who raised more money through the platform. For example, if Clinton raises \$1,000,000 through the platform and Trump raises \$800,000, then the platform donates \$1,600,000 to charity and \$200,000 to Clinton.<sup>1</sup>

One important question is the legality of this mechanism. It turns out that the FEC has ruled on this very question, stating that the mechanism is legal in an advisory opinion issued in 2015 [2]. The ruling was in response to a request by Repledge, a company set up to implement this mechanism that never got off the ground.

However, there are problems with this mechanism. We first consider the following three issues.

- (1) The Apolitical Altruist: suppose that someone who does not care who wins the election wishes to donate \$100 to charity. They could donate to the charity, or they could donate to the candidate who has less money pledged through the platform, thereby sending their \$100 and another \$100 (from a donor to the opposing campaign) to charity. This way, \$200 is donated to charity instead of \$100, but this action distorts the election by depriving the campaign with more donors of \$100 that the campaign “should have” had.

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<sup>1</sup>One particular way that this could work is that every donor specifies which charity they want their money donated to if it is not matched, and the platform donates accordingly.

- (2) The Selfish Charity: similarly, a charity could donate \$100 to the candidate with less money pledged through the platform, selecting themselves as the recipient if their money is matched. The charity gets back the \$100 and also receives an extra \$100 with some positive probability (if the opposing side’s newly-matched \$100 was pledged to the same charity).<sup>2</sup>
- (3) The Large Market Effect: if the platform consumes almost all political contributions, it would be much more difficult for campaigns to spread their messages. While the marginal hundred million dollars have no discernible effect on elections, the first hundred million dollars may.

Interestingly, all three of these issues can be solved or mitigated with a simple change: only sending half of the matched amount to charity. In our example above, \$800,000 would get donated to charity, Clinton would receive \$600,000, and Trump would receive \$400,000. An apolitical altruist is now better off donating directly to the charity of their choice; a selfish charity would lose money via the above scheme; and there would be no chance of the platform causing candidates to have almost no money.

There is one issue, however, that is not so easily solved: the two campaigns may value a dollar differently. This happens when one campaign has raised substantially less than the other, in which case it values a marginal dollar more than the opposing campaign. While in a majority of election cycles the two candidates raise a comparable amount of money, this is often not the case (and was not the case in 2016) [3]. In such a case, one side may be unhappy to have their contributions matched one-to-one, and if one side does not contribute through the platform, the platform becomes useless. In order to attempt to fix this issue, we first need a model of donors’ utilities, so that we may analyze whether or not a given mechanism is incentive-compatible.

## 2. The Donor Model

We will say that there are two political parties, the Democrats and the Republicans. We model the utility of a donor  $i$  as follows:

$$u_i = x_i(\$Dem) + y_i(\$Rep) + z_i(\$Char),$$

where the terms in parentheses are, respectively, the total amount of money that the Democratic candidate receives, the total amount of money that the Republican candidate receives, and the total amount of money donated to charity.<sup>3</sup> If the donor is a Democrat, we have  $x_i, z_i > 0$  and  $y_i < 0$ ; if the donor is a Republican, we have  $y_i, z_i > 0$  and  $x_i < 0$ .

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<sup>2</sup>The details depend on the specific implementation of the mechanism, but in any case the charity gets back more than \$100 in expectation.

<sup>3</sup>This model assumes that there is only one charity, or that there are a number of charities that the donor is indifferent between. This is not necessarily a reasonable assumption, and a refined model might introduce another term that describes the ratio between how much the donor likes their favorite charity and how much the donor likes an average charity (weighted by how many dollars are pledged to the charity by donors supporting the opposite party).

Assume for the moment that donor  $i$  is a Democrat. A natural question to ask about donor  $i$  is how generous they are willing to be when donating to the platform. That is, for every dollar donating to the Republican candidate through the platform, how many of their own dollars are they willing to have matched? Put otherwise, suppose  $i$  is choosing between donating directly to the Democratic candidate, or donating to the platform, so that either their donation goes to the Democratic candidate or their donation gets matched at a ratio of  $\rho$  (i.e.  $\rho$  of their dollars for every dollar pledged to the Republican candidate).<sup>4</sup> What is the largest  $\rho$  that is acceptable to donor  $i$ ?

If donor  $i$  donates a dollar to the Democratic candidate, their utility increases by  $x_i$ . If they donate to the charity, it may increase by  $x_i$ , or it may increase by  $\left(1 + \frac{1}{\rho}\right)z_i - \frac{1}{\rho}y_i$  (because in addition to their dollar being sent to charity, an extra  $\frac{1}{\rho}$  dollars would be redirected from the Republican candidate to charity). Thus, the acceptable values of  $\rho$  are the ones satisfying

$$\left(1 + \frac{1}{\rho}\right)z_i - \frac{1}{\rho}y_i \geq x_i.$$

Solving this equation gives

$$\rho \leq \frac{z_i - y_i}{x_i - z_i}$$

whenever  $x_i > z_i$ . On the other hand, if  $z_i \geq x_i$  then the dichotomy assumed above is a false one: under no circumstances would donor  $i$  prefer to give directly to the Democratic candidate; they would prefer to give their dollar to charity. For this reason, we will assume that  $x_i > z_i$  for all Democratic donors  $i$ , and similarly  $y_i > z_i$  for all Republican donors  $i$ .

If instead  $i$  is a Republican, we can ask the same question: if their donation gets matched at a ratio of  $\rho$  (one of their dollars for every  $\rho$  dollars pledged to the Democratic candidate)<sup>5</sup>, what is the *smallest* acceptable  $\rho$  to donor  $i$ ? In this case, the acceptable values of  $\rho$  are the ones satisfying

$$(\rho + 1)z_i - \rho x_i \geq y_i.$$

Solving this equation gives

$$\rho \geq \frac{y_i - z_i}{z_i - x_i},$$

assuming that  $y_i > z_i$ . This motivates the following definition.

**Definition 2.1.** For each donor  $i$ , we define the quantity  $\rho_i$  to be

$$\rho_i = \frac{z_i - y_i}{x_i - z_i}$$

if  $i$  is a Democratic donor, and

$$\rho_i = \frac{y_i - z_i}{z_i - x_i}$$

if  $i$  is a Republican donor.

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<sup>4</sup>We call  $\rho$  the *matching ratio* of the mechanism.

<sup>5</sup>Note that the way we defined  $\rho$  is the same as above, rather than being defined symmetrically for Republican donors. This is because we are speaking of a mechanism where money gets matched at a ratio of  $\rho$ , so a consistent definition is needed.

Consider a mechanism that matches  $\rho$  Democratic dollars to every one Republican dollar. A Democratic donor  $i$  is willing to participate in the mechanism if  $\rho \leq \rho_i$ . A Republican donor  $i$  is willing to participate in the mechanism if  $\rho \geq \rho_i$ .

We note that because both Democratic and Republican donors have increased utility from donations to charity, we expect Democratic values of  $\rho_i$  to generally be larger than Republican values of  $\rho_i$ , meaning that a Democrat and a Republican should in theory be able to find a value of  $\rho$  that is acceptable to both. For example, if donor 1 (a Democrat) has  $x_1 = 1$ ,  $y_1 = -1$ , and  $z_1 = 0.5$ , and donor 2 (a Republican) has  $x_2 = -1$ ,  $y_2 = 1$ , and  $z_2 = 0.5$ , then we have  $\rho_1 = 3$  and  $\rho_2 = \frac{1}{3}$ , meaning that any  $\rho \in [\frac{1}{3}, 3]$  should be (weakly) preferable to donating directly to the campaigns for both donors.

We now suggest and analyze a natural mechanism that, unlike the one suggested earlier, does not force  $\rho$  to be equal to 1.

### 3. The Market Mechanism

Consider a mechanism (which we will call the market mechanism) that asks each donor  $i$  to state  $\rho_i$ . The mechanism then finds the value of  $\rho$  at which contributions are matched perfectly, sends those contributions to charity (or perhaps half of them, as discussed earlier), and sends unmatched amounts to the respective candidates.

To state this more precisely, define  $F_R : [0, \infty) \rightarrow \mathbb{R}$  as such:  $F_R(x)$  is the total money contributed by Republican donors whose  $\rho_i$  is less than or equal to  $x$ . Put another way,  $F_R(x)$  is the total money contributed by Republican donors who would be happy to have their money matched at  $\rho = x$ . Note that  $F_R$  is a monotone increasing function, with  $F_R(\infty) := \lim_{x \rightarrow \infty} F_R(x)$  equal to the total amount of money contributed through the platform by Republican donors.

Similarly, define  $F_D(x)$  to be the total money contributed by Democratic donors whose  $\rho_i$  is greater than or equal to  $x$ ; that is, money contributed by Democratic donors who would be happy to have their money matched at  $\rho = x$ . Note that  $F_D$  is a monotone decreasing function, with  $F_D(0)$  equal to the amount of money contributed by Democratic donors.

The mechanism then selects  $\rho$  to be the value such that  $F_D(\rho) = \rho F_R(\rho)$ . There is a unique such value of  $\rho$  because  $F_D(\rho)$  monotonically decreases and  $\rho F_R(\rho)$  strictly monotonically increases. This is the correct value to set, because there are  $F_D(\rho)$  and  $F_R(\rho)$  Democratic-pledged and Republican-pledged dollars that donors are happy to have traded at a ratio of  $\rho$  Democratic dollars per Republican dollar, which means that the dollars match exactly. At this rate, the following contributions happen:

- The Democratic candidate receives  $F_D(0) - F_D(\rho) = F_D(0) - \rho F_R(\rho)$  dollars.
- The Republican candidate receives  $F_R(\infty) - F_R(\rho)$  dollars.
- $F_D(\rho) + F_R(\rho) = (1 + \rho)F_R(\rho)$  dollars are contributed to charity.

Note that this value of  $\rho$  is also the value that maximizes charitable contributions. Changing the matching ratio to some smaller value  $x$  makes Republican contributions the bottleneck (so fewer Republican dollars can be matched), so the total amount contributed to charity is

$(1+x)F_R(x)$ , which is less than  $(1+\rho)F_R(\rho)$ . By symmetry, changing the matching ratio to a larger value also decreases the amount donated to charity.

Now we ask, is this mechanism incentive-compatible? That is, is it always in a donor's interest to report their  $\rho_i$  truthfully? Although at first glance the answer appears to be yes (if you tell the truth, your money only gets matched if you are happy with the value  $\rho$ , so there is no point to lie), this intuition misses the fact that by reporting some value that is not  $\rho_i$ , a donor can influence the value of  $\rho$ . This is an important point even under large-market assumptions: while the amount that  $\rho$  will change from one donor misreporting will decrease to zero, the effect that the change will have on the resulting contributions will increase.

To analyze whether the mechanism is incentive-compatible, consider a donor  $i$  and without loss of generality assume  $i$  is a Democrat. Suppose that by lying, donor  $i$  can change the matching ratio to  $\rho' = \rho + \delta$ .<sup>6</sup> Note that while doing this changes the function  $F_D$ , it does not change  $F_D(0)$  and does not change the function  $F_R$ . This means that at the matching ratio  $\rho'$ , the Democratic candidate receives  $F_D(0) - \rho'F_R(\rho')$  dollars; the Republican candidate receives  $F_R(\infty) - F_R(\rho')$  dollars; and  $(1+\rho')F_R(\rho')$  dollars are contributed to charity. Thus, the change in donor  $i$ 's utility from misreporting is equal to

$$\begin{aligned}\Delta u_i &= x_i(\rho F_R(\rho) - \rho' F_R(\rho')) + y_i(F_R(\rho) - F_R(\rho')) + z_i((1+\rho')F_R(\rho') - (1+\rho)F_R(\rho)) \\ &= (z_i - x_i)(\rho' F_R(\rho') - \rho F_R(\rho)) + (z_i - y_i)(F_R(\rho') - F_R(\rho)).\end{aligned}$$

It makes sense for donor  $i$  to lie if and only if  $\Delta u_i > 0$ , so we wish to know when the above quantity is positive. Assuming that  $\delta \approx 0$  and that  $F_R$  is continuous,<sup>7</sup> we can write

$$F_R(\rho') - F_R(\rho) \approx \delta F'_R(\rho)$$

and

$$\rho' F_R(\rho') - \rho F_R(\rho) = \rho(F_R(\rho') - F_R(\rho)) + \delta F_R(\rho') \approx \delta(\rho F'_R(\rho) + F_R(\rho)).$$

This lets us write

$$\Delta u_i \approx \delta((z_i - x_i)(\rho F'_R(\rho) + F_R(\rho)) + (z_i - y_i)F'_R(\rho)).$$

We consider two cases. First, assume that  $\delta$  is positive. This means that the donor lies in a way that increases the matching ratio. This can only be done by increasing the value of  $F_D$  around  $\rho$ , which is only possible to do if  $\rho_i$  is smaller than  $\rho$ , by reporting a value that is larger than  $\rho$ .

In this case,  $\Delta u_i$  is positive when  $(z_i - y_i)F'_R(\rho) > (x_i - z_i)(\rho F'_R(\rho) + F_R(\rho))$ . Assuming that  $x_i > z_i$  (as discussed previously), this is the case if and only if  $\rho_i > \rho + \frac{F_R(\rho)}{F'_R(\rho)}$ . But this is impossible, since donor  $i$  cannot increase the matching ratio by misreporting if  $\rho_i > \rho$ .

Now assume that  $\delta$  is negative. This means that the donor's lie decreases the value of  $F_D$  around  $\rho$ , which means that  $\rho_i$  is larger than  $\rho$ , but the donor reports a value that is smaller than  $\rho$ .

In this case,  $\Delta u_i$  is positive when  $(z_i - y_i)F'_R(\rho) < (x_i - z_i)(\rho F'_R(\rho) + F_R(\rho))$ . Assuming that  $x_i > z_i$ , this is the case if and only if  $\rho_i < \rho + \frac{F_R(\rho)}{F'_R(\rho)}$ . This answers our question:

<sup>6</sup>We will treat  $\delta$  as being very small; this makes sense if the market is large.

<sup>7</sup>This is not immediately obvious as an acceptable assumption, since donors may round their values of  $\rho_i$  e.g. to the nearest hundredth; however, since in practice the function  $F_R$  is not known to donor  $i$ , the donor is really dealing with expected values, and those can be assumed to be continuous.

**Theorem 3.1.** Under reasonable assumptions and perfect information, in the market mechanism a Democratic donor should report the true value  $\rho_i$  unless  $\rho_i \in \left[\rho + \frac{F_R(\rho)}{F'_R(\rho)}\right)$ , in which case the donor should report a value that is less than  $\rho$ .

Similarly, there is an interval close to  $\rho$  such that for Republican donors with values of  $\rho_i$  in this interval it makes sense to lie and state a value that is greater than  $\rho$ . Unfortunately, this means that unless  $F_D$  and  $F_R$  are non-differentiable at  $\rho$ , it necessarily makes sense for some donors to drop out of the mechanism (i.e. report 0 in the case of Democrats and  $\infty$  in the case of Republicans). This lets us conclude the following:

**Corollary 3.2.** Under reasonable assumptions and perfect information, the market mechanism unravels.

We note that when the curves  $F_D$  and  $F_R$  are not known by every player, as it is still in a donor's interest to misreport their value if it is possible to do so in a way that makes  $\mathbb{E}[\Delta u_i]$  positive. We have no reason to believe the mechanism becomes strategy-proof under imperfect information, given that it is quite far from strategy-proof under perfect information.

However, it should be noted that the original mechanism proposed in Section 1 is actually a modification of the market mechanism: it is precisely the market mechanism where Democrats must choose to report either  $\rho_i = 0$  or  $\rho_i = 1$ , and Republicans must choose to report either  $\rho_i = 1$  or  $\rho_i = \infty$ . Indeed, participating in the mechanism means reporting  $\rho_i = 1$ , whereas donating directly to one's preferred candidate is the same as participating in the market mechanism but reporting  $\rho_i = 0$  (in the case of Democrats) or  $\rho_i = \infty$  (in the case of Republicans). The reason that this mechanism does not unravel is that the functions  $F_R$  and  $F_D$  are not continuous at  $\rho = 1$  (intuitively, think of  $F'_R(1)$  as being infinite in the interval established in Theorem 3.1). This raises the following question.

**Question 3.3.** Is it possible to modify the market mechanism by substantially restricting the values of  $\rho_i$  that donors are allowed to report, in a way that prevents the mechanism from unraveling?

We leave this question open.

## 4. The Donor-Matching Mechanism

There are a few ways in which the market mechanism is very natural. First, it finds a particular value of  $\rho$  at which matches happen, and all matches happen at that ratio. Second, the value of  $\rho$  satisfies not one but two desired properties: the amount of money available for matching on either side at  $\rho$  matches exactly (this is how  $\rho$  is defined), but also  $\rho$  is the value that maximizes the amount of money given to charity.

However, it is possible to send even more money to charity by giving up the desired property of having one universal  $\rho$ . Indeed, consider the following example: there are three Democratic donors  $1_D, 2_D, 3_D$  and three Republican donors  $1_R, 2_R, 3_R$ , with  $\rho_{1_D} = \rho_{1_R} = 0.9$ ,  $\rho_{2_D} = \rho_{2_R} = 1$ , and  $\rho_{3_D} = \rho_{3_R} = 1.1$ . All three have a budget of \$100. The matching mechanism would set  $\rho = 1$  and send \$400 to charity, with the Democratic candidate getting \$100 from donor  $1_D$  and the Republican candidate getting \$100 from donor  $3_R$ .

We could instead pair up donor  $1_D$  with  $1_R$ ,  $2_D$  with  $2_R$ , and  $3_D$  with  $3_R$ , having them matched at the  $\rho_i$ 's they report. The result (rounding to the nearest ten dollars) is that \$580 is sent to charity and each candidate gets \$10. From the perspective of matching as many contributions as possible, this does much better.

Suppose that our goal is to maximize the amount of money that gets sent to charity. We may translate this goal into a linear program. Let  $i$  index Democratic donors and  $j$  index Republican donors. Let  $P_i$  be the amount pledged to the platform by (Democratic) donor  $i$  and  $P_j$  be the amount pledged by (Republican) donor  $j$ . Let  $\rho_i$  and  $\rho_j$  be as in Definition 2.1 (we assume that each  $\rho_j$  is strictly positive and each  $\rho_i$  is finite). Our LP is as follows.

$$\begin{array}{ll}
\text{Variables: } x_{ij}, y_{ij} & \forall i, j \\
\text{Maximize } \sum_{i,j} x_{ij} + y_{ij} & \text{subject to} \\
\sum_j x_{ij} \leq P_i & \forall i \\
\sum_i y_{ij} \leq P_j & \forall j \\
\rho_j y_{ij} \leq x_{ij} \leq \rho_i y_{ij} & \forall i, j \\
x_{ij}, y_{ij} \geq 0 & \forall i, j
\end{array}$$

Here,  $x_{ij}$  is the variable for how much of the money pledged by donor  $i$  is matched against a donation by donor  $j$ , and  $y_{ij}$  is the variable for how much of the money pledged by donor  $j$  is matched against a donation by donor  $i$ .

It is unclear if there is an algorithm to solve this LP (which is a generalization of the maximum matching problem) in a more efficient way than simply using the general algorithm for solving LPs. We can nevertheless make some observations about the optimal solution to this LP. First define  $\rho_{ij} := \frac{x_{ij}}{y_{ij}}$ , with  $\rho_{ij}$  remaining undefined if  $x_{ij}, y_{ij} = 0$ . Note that  $\rho_j \leq \rho_{ij} \leq \rho_i$  for all  $i, j$  for which  $\rho_{ij}$  is defined. We also define the following class of feasible LP solutions.

**Definition 4.1.** Given a feasible solution to the LP, define  $\rho_D$  to be the largest value of  $\rho_i$  such that  $\sum_j x_{ij} < P_i$  (i.e. not all of  $i$ 's money goes to charity) and define  $\rho_R$  to be the smallest value of  $\rho_j$  such that  $\sum_i y_{ij} < P_j$ . A feasible solution to the LP is called *canonical* if it has the following structure:

- For a Democratic donor  $i$ , if  $\rho_i < \rho_D$  then each  $x_{ij}$  equals 0 (i.e. all of  $i$ 's money goes to the Democratic candidate).
- For a Republican donor  $j$ , if  $\rho_j > \rho_R$  then each  $y_{ij}$  equals 0 (i.e. all of  $j$ 's money goes to the Republican candidate).

Note that the market mechanism is a canonical solution with  $\rho_D = \rho_R = \rho$  (with  $\rho$  as defined earlier). It can alternatively be described as the optimal LP solution with the additional constraint that  $\rho_D = \rho_R$ .

Note also that  $\rho_D$  and  $\rho_R$  for a canonical solution describe the efficiency of the mechanism. The smaller  $\rho_D$  and the larger  $\rho_R$ , the more people the mechanism successfully paired. On

the other hand, when  $\rho_D$  and  $\rho_R$  are close together, comparatively many contributions are left unpaired. In fact, we can write the value of the objective function of a canonical solution as

$$\sum_{i,j} x_{ij} + y_{ij} = \sum_{i:\rho_i > \rho_D} P_i + \sum_{j:\rho_j < \rho_R} P_j + \sum_{i:\rho_i = \rho_D} \sum_j x_{ij} + \sum_{j:\rho_j = \rho_R} \sum_i y_{ij}.$$

That is, up to donors  $i$  with  $\rho_i$  exactly equal to  $\rho_D$  and donors  $j$  with  $\rho_j$  exactly equal to  $\rho_R$ , the amount donated to charity (and to the candidates) is determined by  $\rho_D$  and  $\rho_R$ .

We now prove the following claim.

**Claim 4.2.** There is an optimal solution to the LP that is canonical.

*Proof.* Consider the optimal solution to the LP that minimizes the extent to which the above structure is violated, i.e. the optimal solution that minimizes

$$\sum_{i:\rho_i < \rho_D} \sum_j x_{ij} + \sum_{j:\rho_j > \rho_R} \sum_i y_{ij}.$$

(The fact that this minimum exists follows from the fact that the space of optimal solutions is compact and the above quantity is a continuous function of the setting of variables.) If this quantity is nonzero, without loss of generality assume the first term is nonzero. Then there exists a Democratic donor  $i$  with  $\rho_i < \rho_D$  and a Republican donor  $j$  such that  $x_{ij} > 0$ . Let  $i'$  be the Democratic donor with  $\rho_{i'} = \rho_D$  such that  $\sum_j x_{i'j} < P_{i'}$ . Consider the LP solution that pairs  $j$  with  $i'$  instead for the same amount of money, setting  $\rho_{i'j}$  equal to the current value of  $\rho_{ij}$ . (This is legal because if  $\rho_{ij} \leq \rho_i$  then  $\rho_{ij} \leq \rho_{i'}$ .) But this LP solution (which achieves the same value of the objective function) has a strictly smaller value for the above quantity, yielding a contradiction.

The fact that  $\rho_D \leq \rho_R$  follows from the fact that if  $\rho_D > \rho_R$ , we can pair up the  $i$  satisfying  $\rho_i = \rho_D$  and  $\sum_j x_{ij} < P_i$  with the  $j$  satisfying  $\rho_j = \rho_R$  and  $\sum_i y_{ij} < P_j$ , creating a solution with a larger value for the objective function.  $\square$

We also show the following fact.

**Claim 4.3.** Consider a canonical optimal solution to the LP. Let  $i$  be a Democratic donor with  $\rho_i \geq \rho_R$ . Then for all  $j$  such that  $x_{ij} > 0$  (i.e. some of  $i$ 's money is matched to  $j$ 's money), we have  $\rho_{ij} = \rho_j$ , and either  $\rho_j > \rho_D$  or  $\rho_i = \rho_j = \rho_D = \rho_R$ . Similarly, let  $j$  be a Republican donor with  $\rho_j \leq \rho_D$ . Then for all  $i$  such that  $y_{ij} > 0$ , we have  $\rho_{ij} = \rho_i$ , and either  $\rho_i < \rho_R$  or  $\rho_i = \rho_j = \rho_D = \rho_R$ .

*Proof.* We first show that in the first part of the claim, we have  $\rho_{ij} = \rho_j$ . Indeed, suppose for contradiction that  $\rho_{ij} > \rho_j$ . Consider a slightly modified feasible solution that slightly increases  $\rho_{ij}$  while keeping  $y_{ij}$  the same, thus slightly decreasing  $x_{ij}$ . Now donor  $i$  has some unmatched money. Match this money with money from the donor  $j'$  satisfying  $\rho_{j'} = \rho_R$  and  $\sum_i y_{ij'} < P_{j'}$ . (For a small enough change in  $\rho_{ij}$  this is necessarily possible.) This increases the value of the objective function, a contradiction.

Now, suppose that  $\rho_j \leq \rho_D$ . By the claim proven in the previous paragraph applied symmetrically to Republican donors, we have  $\rho_{ij} = \rho_i$ . On the other hand we just showed that  $\rho_{ij} = \rho_j$ . This means that  $\rho_i = \rho_j$ , from which it also follows that  $\rho_i = \rho_j = \rho_D = \rho_R$  (since  $\rho_j \leq \rho_D$  but  $\rho_i \geq \rho_D$ , and since  $\rho_i \geq \rho_R$  but  $\rho_j \leq \rho_R$ ).  $\square$

**Corollary 4.4.** Every value of  $\rho_{ij}$  is between  $\rho_D$  and  $\rho_R$ , inclusive. In fact, every such value is between  $\rho_D$  and  $\rho_R$ , exclusive, unless  $\rho_D = \rho_R$ .

*Proof.* If  $\rho_i \geq \rho_R$ , this follows immediately. If  $\rho_j \leq \rho_D$ , this follows immediately. If  $\rho_i < \rho_R$  and  $\rho_j > \rho_D$ , then we also have this result because  $\rho_j \leq \rho_{ij} \leq \rho_i$ .  $\square$

One way to look at what we have proven is this: for some constants,  $\rho_D$  for a canonical optimal solution is much smaller than  $\rho_R$ ; this is good because more money gets sent to charity. However, if  $\rho_D$  is close in value to  $\rho_R$ , another interesting (and perhaps desirable) property arises, namely *predictability*: the rate at which donors' money will be matched will all lie in the narrow interval between  $\rho_D$  and  $\rho_R$ .

We end with the obvious open question, namely that of strategy-proofness.

**Question 4.5.** Is there a way of choosing a (perhaps canonical) optimal solution in a way that is strategy-proof (i.e. each Democratic donor  $i$  is incentivized to report the true value of  $\rho_i$ , and similarly for Republican donors  $j$ )?

Note that the fact that the amount of money donated to charity and to each candidate can (almost) be written as a function of  $\rho_D$  and  $\rho_R$  provides a possible pathway to approaching this question. Instead of analyzing an unwieldy LP, perhaps it is easier to analyze the effect of lying on the values  $\rho_D$  and  $\rho_R$  of an optimal canonical solution.

## References

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